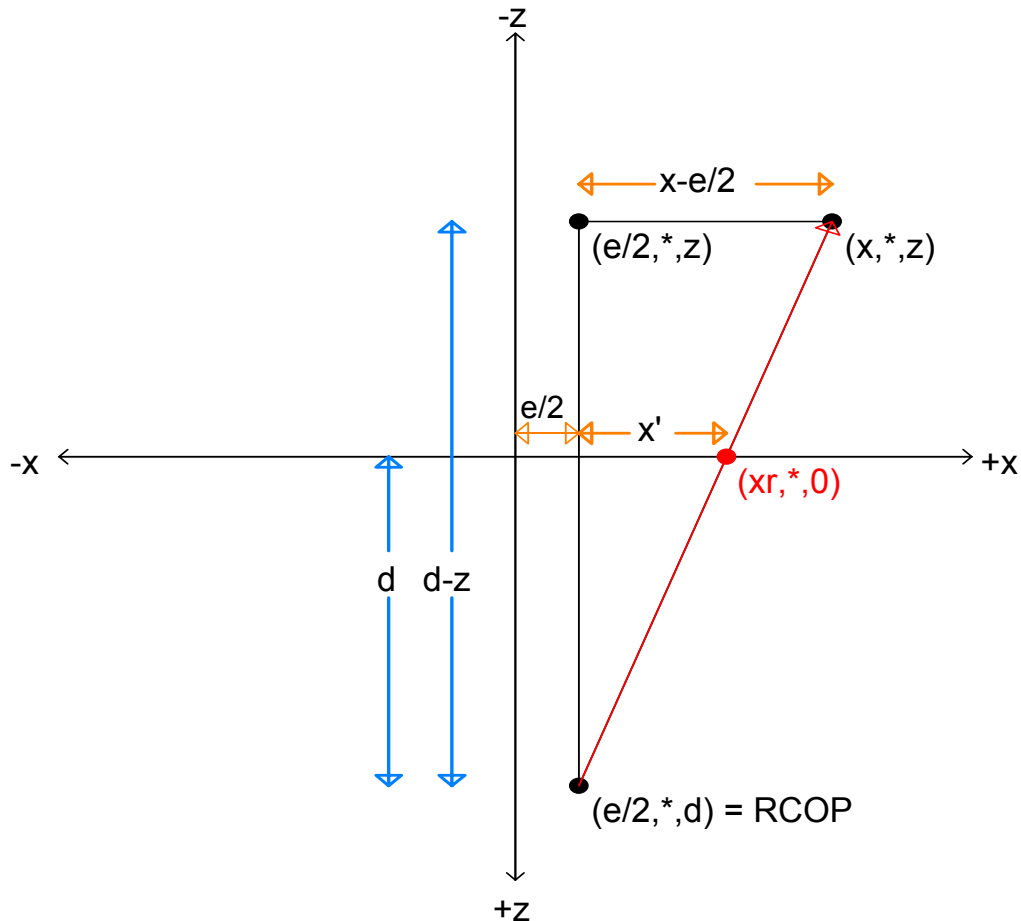


Deriving the Formula for Perspective Projection from the Right Eye.



Notes: Right hand coordinates

Orthographic view down from $+y$.

RCOP = right center of projection, d units from the plane.

$e/2 = \frac{1}{2}$ eye separation, $*$ above means omitted.

≈ 1.1 inches

$$x_r = x' + e/2$$

$$x'/(x-e/2) = d / (d-z)$$

construction

similar triangles

Note that z is a negative value in the diagram. The length is $d-z$, not $d+z$

$$x' = (x - e/2) * d / (d - z) \quad \text{multiplication}$$

$$x_{r-e/2} = (x - e/2) * d / (d - z) \quad \text{substitution}$$

$$x_r = (x - e/2) * d / (d - z) + e/2 * [(d - z) / (d - z)]$$

addition and multiplication by 1

$$x_r = (d * x - e * z / 2) / (d - z) \quad \text{cancel terms}$$

$$x_l = (d * x + e * z / 2) / (d - z)$$

left eye: the sign for e changes

$$y_b = d * y / (d - z)$$

there is no shift for the eye position
in the y direction

$$x_r = (dx - ez/2) / (d - z)$$

Other notation

$$x_l = (dx + ez/2) / (d - z)$$

$$y_b = dy / (d - z)$$

Second Approach For The Right Eye

Derivation using Points and Vectors

$$\begin{aligned} (x_r, y_b, 0) &= s * [(x, y, z) - (e/2, 0, d)] + (e/2, 0, d) \\ \text{projected point} &= \text{Scaled vector from RCOP} + \text{RCOP} \end{aligned}$$

RCOP = right eye center of projection

$$\begin{aligned} 0 &= s * (z - d) + d && \text{Equation for the z coordinate} \\ s &= d / (d - z) && \text{Solving for the scale factor s.} \end{aligned}$$

$$\begin{aligned} x_r &= s * (x - e/2) + e/2 && \text{Equation for the x coordinate} \\ y_b &= s * y && \text{Equation for the y coordinate} \end{aligned}$$

$$\begin{aligned} x_r &= (d * x - e * z / 2) / (d - z) && \text{Substitute for s and simplify} \\ y_b &= d * y / (d - z) && \text{Note } y_b \text{ is the same for both eyes} \\ x_l &= (d * x + e * z / 2) / (d - z) && \text{The sign changes for e using the} \\ &&& \text{LCOP} \end{aligned}$$